

Chemical fluctuations in high-energy nuclear collisions

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Fluctuations of the chemical composition of the hadronic system produced in nuclear collisions are discussed using the Φ -measure which has been earlier applied to study the transverse momentum fluctuations. The measure is expressed through the moments of the multiplicity distribution and then the properties of Φ are discussed within a few models of multiparticle production. A special attention is paid to the fluctuations in the equilibrium ideal quantum gas. The system of kaons and pions, which is particularly interesting from the experimental point of view, is discussed in detail.

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There are many sorts of hadrons produced in high energy collisions. The ratios of multiplicities of particles of given species to the total particle number characterize the chemical composition of the collision final state. The composition is expected to reflect the collisions dynamics. Generation of the quark-gluon plasma in heavy-ion collisions was argued long ago [1] to enhance the strange particle production. While the significant strangeness yield enhancement has been experimentally observed in the central nucleus-nucleus collisions at CERN SPS (see the data compilation [2] and the recent review [3]), it is a matter of hot debate whether the observation can be indeed treated as a plasma signal. The strange hadron abundance is naturally described within the models assuming the plasma occurrence at the early collision stage (see e.g. [4,5]) but the models, which neglect such a possibility (see e.g. [6,7] or the review [8]), can be also tuned to agree with the experimental data. Thus, it would be desirable to go beyond the average particle numbers and see whether the strangeness enhancement in the central heavy-ion collisions is accompanied with the qualitative change of the strangeness yield fluctuations. The equilibrium quark-gluon plasma scenario is obviously expected to lead to the smaller fluctuations than the nonequilibrium cascade-like hadron models but the specific calculations are needed to quantify such a prediction. Anyhow, it seems to be really interesting to study the strangeness yield fluctuations on the event-by-event basis. However, we immediately face the difficulty how to quantitatively measure the fluctuations in the events of very different multiplicity. The problem appears to be of more general nature.

There are several interesting proposals to use fluctuations as a potential source of valuable information on the collision dynamics. If the hadronic system produced in the collision is in the thermodynamical equilibrium, the temperature and multiplicity fluctuations have been argued to determine, respectively, the heat capacity [9,10] and compressibility [11] of the hadronic matter at freeze-out. An extensive discussion of the equilibrium fluctuations can be found in [12]. In the experimental realization of such ideas one has to disentangle however the ‘dynamical’ fluctuations of interest from the ‘trivial’ geometrical ones due to the impact parameter variation. The latter fluctuations are very sizable and dominate the fluctuations of all extensive event characteristics such as multiplicity or transverse energy. The variation of the impact parameter can also influence the fluctuations of the intensive quantities e.g. the temperature.

A specific solution to the problem was given in our paper [13], where we introduced the measure of fluctuations or correlations which has been later called Φ . It is constructed in such a way that Φ is exactly the same for nucleon-nucleon (N-N) and nucleus-nucleus (A-A) collisions if the A-A collision is a simple superposition of N-N interactions. On the other hand, Φ equals zero when the correlations are absent in the collision final state. The method proposed in [13] has been recently applied to the NA49 experimental data. The fluctuations of transverse momentum found in the central Pb-Pb collisions at 158 GeV per nucleon have appeared to be surprisingly small [14,15]. It has been also claimed [15] that the correlations, which are of short range in the momentum space, are responsible for the nonzero

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positive value of Φ_{p_T} being observed. Our calculations of Φ_{p_T} in the equilibrium ideal gas show [16] that Φ_{p_T} is positive for bosons, negative for fermions and zero for classical particles. When the hadronic system at freeze-out is identified with the pion gas, the calculated Φ_{p_T} slightly overestimates the experimental value [15] but the inclusion of the pions which come from the resonance decays removes the discrepancy. An interesting analysis of the p_T -fluctuations within the so-called non-extensive statistics is given in [17].

The theoretical analysis of the result [14,15] has provided a new insight into the collision dynamics. It has been argued [18] within the UrQMD model that the secondary scatterings are responsible for the dramatic correlation loss in the central collisions of heavy-ions. This conclusion however seems to contradict the results of the analysis [19] where the LUCIAE event generator has been used and the rescatterings are shown to reduce insignificantly the p_T -correlations measured by Φ . While the effect of the secondary interactions needs to be clarified, the smallness of the fluctuations observed in the central heavy-ion collisions [14,15] is a very restrictive test of the collision models. The so-called random walk model is ruled out because it gives much stronger correlations in A-A than N-N case [22]. The same holds [19] for the LUCIAE event generator when the jet production and/or the string clustering is taken into account even at a rather moderate rate. On the other hand, the quark-gluon string model seems to pass the test successfully [21].

As argued in [22], the measure Φ can be also applied to study the fluctuations of chemical composition of the hadronic system produced in the nuclear collisions. The chemical fluctuations seem to be even more interesting than those of the kinematical variables such as p_T . The final state momentum distribution of hadrons characterizes the system at the moment of freeze-out, while the system chemical composition is fixed at the earlier evolution stage - the chemical freeze-out when the secondary *inelastic* interactions are no longer effective. The total strangeness yield presumably saturates even earlier and the subsequent interactions are mostly responsible for the strangeness redistribution among hadron species.

The NA49 Collaboration plans to study the chemical fluctuations in heavy-ion collisions at CERN SPS [23]. Since the Φ -measure will be used in these studies, it is desirable to better understand the properties of Φ when applied to the chemical fluctuations. This is the aim of our note. At the beginning we express Φ through the commonly used moments of the multiplicity distribution and analyze the result within several models of the distribution. Then, we compute the Φ -measure for case of the two-component ideal quantum gas in equilibrium. The system of kaons and pions is discussed in detail. In particular, the role of resonances is analysed.

Let us first introduce the measure Φ which describes the correlations (or fluctuations) of a single particle variable x such as the particle energy or transverse momentum. As observed in [22], x can also characterize the particle sort. Then, $x = 1$ if the particle is of the sort of interest, say the particle is strange, and $x = 0$ if the particle is *not* of this sort, it is a non-strange particle. We define the single-particle variable $z \stackrel{\text{def}}{=} x - \bar{x}$ with the overline denoting averaging over a single particle inclusive distribution. In the case of the chemical fluctuations, \bar{x} is the probability (averaged over events and particles) that a produced particle is of the sort of interest, say it is strange. One easily observes that $\bar{z} = 0$. We now introduce the event variable Z , which is a multiparticle analog of z , defined as $Z \stackrel{\text{def}}{=} \sum_{i=1}^N (x_i - \bar{x})$, where the summation runs over particles from a given event. By construction $\langle Z \rangle = 0$, where $\langle \dots \rangle$ represents averaging over events. Finally, the Φ -measure is defined in the following way

$$\Phi \stackrel{\text{def}}{=} \sqrt{\frac{\langle Z^2 \rangle}{\langle N \rangle}} - \sqrt{\bar{z}^2}. \quad (1)$$

We compute Φ for the system of particles of two sorts, a and b , e.g. strange and non-strange hadrons. $x_i = 1$ when i -th particle is of the a type and $x_i = 0$ otherwise. The inclusive average of x and x^2 read

$$\bar{x} = \sum_{x=0,1} x P_x = P_1, \quad \overline{x^2} = \sum_{x=0,1} x^2 P_x = P_1,$$

where P_1 is the probability (averaged over particles and events) that a produced particle is of the a sort. Thus,

$$P_1 = \frac{\langle N_a \rangle}{\langle N_a \rangle + \langle N_b \rangle},$$

with N_a and N_b being the numbers of particles a and b , respectively, in a single event. One immediately finds that $\bar{z} = 0$ while

$$\overline{z^2} = P_1 - P_1^2 = \frac{\langle N_a \rangle \langle N_b \rangle}{\langle N \rangle^2}, \quad (2)$$

where $N = N_a + N_b$ is the multiplicity of all particles a and b in a single event.

Since the event variable Z equals $N_a - \bar{x}N$, one gets

$$\begin{aligned}\langle Z \rangle &= \langle N_a \rangle - \bar{x} \langle N \rangle = 0 , \\ \langle Z^2 \rangle &= \langle N_a^2 \rangle - 2\bar{x} \langle N_a N \rangle + \bar{x}^2 \langle N^2 \rangle .\end{aligned}$$

The latter equation gives

$$\langle Z^2 \rangle \langle N \rangle^2 = \langle N_b \rangle^2 \langle N_a^2 \rangle + \langle N_a \rangle^2 \langle N_b^2 \rangle - 2 \langle N_a \rangle \langle N_b \rangle \langle N_a N_b \rangle ,$$

which can be rewritten as

$$\begin{aligned}\frac{\langle Z^2 \rangle}{\langle N \rangle} &= \frac{\langle N_b \rangle^2}{\langle N \rangle^3} (\langle N_a^2 \rangle - \langle N_a \rangle^2) + \frac{\langle N_a \rangle^2}{\langle N \rangle^3} (\langle N_b^2 \rangle - \langle N_b \rangle^2) \\ &\quad - 2 \frac{\langle N_a \rangle \langle N_b \rangle}{\langle N \rangle^3} (\langle N_a N_b \rangle - \langle N_a \rangle \langle N_b \rangle) .\end{aligned}\tag{3}$$

The fluctuation measure Φ is completely determined by eqs. (2, 3). So, let us consider its properties within three simple models of the multiplicity distribution.

1) The distributions of particles a and b are poissonian and independent from each other i.e.

$$\langle N_i^2 \rangle - \langle N_i \rangle^2 = \langle N_i \rangle ,\tag{4}$$

$$\langle N_a N_b \rangle = \langle N_a \rangle \langle N_b \rangle ,$$

where $i = a, b$. One easily notices that $\Phi = 0$ in this case.

2) The particles a and b are assumed to be correlated in such a way that there are *no* chemical fluctuations in the system. The event chemical composition, which is fully characterized (for a two component system) by the ratio N_a/N , is assumed to be strictly independent of the event multiplicity. Then, $N_a = \alpha N$ and $N_b = (1 - \alpha) N$ with α being a constant smaller than unity. Since N_a , N_b and N are the integer numbers, α has to be a rational fraction. Then, we have

$$\langle N_a \rangle = \alpha \langle N \rangle , \quad \langle N_b \rangle = (1 - \alpha) \langle N \rangle ,$$

$$\langle N_a^2 \rangle - \langle N_a \rangle^2 = \alpha^2 (\langle N^2 \rangle - \langle N \rangle^2) , \quad \langle N_b^2 \rangle - \langle N_b \rangle^2 = (1 - \alpha)^2 (\langle N^2 \rangle - \langle N \rangle^2) ,$$

$$\langle N_a N_b \rangle - \langle N_a \rangle \langle N_b \rangle = \alpha(1 - \alpha) (\langle N^2 \rangle - \langle N \rangle^2) .$$

Consequently, $\langle Z^2 \rangle = 0$ and

$$\Phi = -\sqrt{\frac{\langle N_a \rangle \langle N_b \rangle}{\langle N \rangle^2}} = -\sqrt{\alpha(1 - \alpha)} .\tag{5}$$

One sees that the Φ -measure is negative (but larger than $-1/2$) when the chemical fluctuations vanish in the system.

3) The particles a and b are identified with the positively and, respectively, negatively charged hadrons. Then, the charge conservation leads to the strict correlation of the particle numbers:

$$N_+ - N_- = Q ,$$

where Q is the electric charge of the system. In this case we have

$$\langle N_+ \rangle = \langle N_- \rangle + Q ,$$

$$\langle N_+^2 \rangle - \langle N_+ \rangle^2 = \langle N_-^2 \rangle - \langle N_- \rangle^2 ,$$

$$\langle N_+ N_- \rangle - \langle N_+ \rangle \langle N_- \rangle = \langle N_-^2 \rangle - \langle N_- \rangle^2 .$$

Therefore,

$$\frac{\langle Z^2 \rangle}{\langle N \rangle^2} = \frac{(\langle N_- \rangle + Q) \langle N_- \rangle}{\langle N \rangle^2} ,$$

$$\frac{\langle Z^2 \rangle}{\langle N \rangle} = \frac{Q^2}{\langle N \rangle^3} (\langle N_-^2 \rangle - \langle N_- \rangle^2) .$$

When $Q = 0$ we reproduce the result (5) corresponding to $\alpha = 1/2$.

After the illustrative examples let us compute Φ for the equilibrium gas which is a mixture of the particles a and b . Then,

$$\langle N_i \rangle = \lambda_i \frac{\partial}{\partial \lambda_i} \ln \Xi(V, T, \lambda_a, \lambda_b) , \quad (6)$$

$$\langle N_a N_b \rangle - \langle N_a \rangle \langle N_b \rangle = \lambda_a \lambda_b \frac{\partial^2}{\partial \lambda_b \partial \lambda_a} \ln \Xi(V, T, \lambda_a, \lambda_b) ,$$

$$\langle N_i^2 \rangle - \langle N_i \rangle^2 = \left(\lambda_i \frac{\partial}{\partial \lambda_i} \right)^2 \ln \Xi(V, T, \lambda_a, \lambda_b) ,$$

where $\Xi(V, T, \lambda_a, \lambda_b)$ is the grand canonical partition function with V , T and λ_i denoting, respectively, the system volume, temperature and the fugacity which is related to the chemical potential μ_i as $\lambda_i = e^{\beta \mu_i}$ with $\beta \equiv T^{-1}$.

When the gas of interest is the mixture of the two ideal quantum gases, the partition function is [24]

$$\ln \Xi(V, T, \lambda_a, \lambda_b) = \pm g_a V \int \frac{d^3 p}{(2\pi)^3} \ln [1 \pm \lambda_a e^{-\beta E_a}] \pm g_b V \int \frac{d^3 p}{(2\pi)^3} \ln [1 \pm \lambda_b e^{-\beta E_b}] , \quad (7)$$

where g_i denotes the number of the particle internal degrees of freedom; $E_i \equiv \sqrt{m_i^2 + \mathbf{p}^2}$ is the particle energy with m_i and \mathbf{p} being its mass and momentum; the upper sign is for fermions while the lower one for bosons.

Substituting the ideal gas partition function (7) into eqs. (6), one easily finds

$$\langle N_i \rangle = g_i V \int \frac{d^3 p}{(2\pi)^3} \frac{1}{\lambda_i^{-1} e^{\beta E_i} \pm 1} , \quad (8)$$

$$\langle N_a N_b \rangle = \langle N_a \rangle \langle N_b \rangle ,$$

$$\langle N_i^2 \rangle - \langle N_i \rangle^2 = g_i V \int \frac{d^3 p}{(2\pi)^3} \frac{\lambda_i^{-1} e^{\beta E_i}}{(\lambda_i^{-1} e^{\beta E_i} \pm 1)^2} ,$$

where, as presviously, the index i labels the particles of the type a or b . It is worth noting that the system volume V which enters eqs. (8) cancels out in the final expression of Φ . Therefore, the measure Φ is, as expected, an intensive quantity. One observes in eqs. (8) that

$$\langle N_i^2 \rangle - \langle N_i \rangle^2 < \langle N_i \rangle$$

for fermions,

$$\langle N_i^2 \rangle - \langle N_i \rangle^2 > \langle N_i \rangle$$

for bosons, and

$$\langle N_i^2 \rangle - \langle N_i \rangle^2 = \langle N_i \rangle$$

in the classical limit where $\lambda_i^{-1} \gg 1$. Therefore, one finds from eqs. (2, 3) that

$$\frac{\langle Z^2 \rangle}{\langle N \rangle} < \overline{z^2} \quad \text{and} \quad \Phi < 0$$

when the particles a and b are fermions,

$$\frac{\langle Z^2 \rangle}{\langle N \rangle} > \overline{z^2} \quad \text{and} \quad \Phi > 0$$

when the particles a and b are bosons, and

$$\frac{\langle Z^2 \rangle}{\langle N \rangle} = \overline{z^2} \quad \text{and} \quad \Phi = 0$$

when the particles of both types can be treated as classical. If the particles a and b are of different statistics, the sign of Φ is determined by the sign of the expression

$$\langle N_a \rangle^2 (\langle N_b^2 \rangle - \langle N_b \rangle^2 - \langle N_b \rangle) + \langle N_b \rangle^2 (\langle N_a^2 \rangle - \langle N_a \rangle^2 - \langle N_a \rangle) ,$$

which is either positive or negative depending of the particle masses, their chemical potentials, and the numbers of the internal degrees of freedom.

If all particles are massless and their chemical potentials vanish, the calculations can be performed analytically. In this case eqs. (8) give

$$\langle N_i \rangle = \frac{g_i \zeta(3)}{\pi^2} \binom{3/4}{1} VT^3 \cong g_i \binom{0.09}{0.12} VT^3 ,$$

$$\langle N_i^2 \rangle - \langle N_i \rangle^2 = \frac{g_i}{6} \binom{1/2}{1} VT^3 \cong g_i \binom{0.08}{0.17} VT^3 ,$$

where $\zeta(x)$ is the Riemann zeta function ($\zeta(3) \cong 1.202$); as previously the upper case is for fermions and the lower one for bosons. If the particles a and b are both fermions or both bosons, eqs. (2, 3) get the form

$$\overline{z^2} = \frac{g_a g_b}{(g_a + g_b)^2} , \tag{9}$$

$$\frac{\langle Z^2 \rangle}{\langle N \rangle} = \frac{\pi^2}{6\zeta(3)} \frac{g_a g_b}{(g_a + g_b)^2} \binom{2/3}{1} ,$$

and consequently

$$\Phi \cong \binom{-0.045}{0.170} \frac{\sqrt{g_a g_b}}{g_a + g_b} .$$

Let us now consider the fluctuations in the system of pions and kaons. To be specific, the particles a are identified with π^- while the particles b with K^+ or K^- . As we will see, the fluctuations in the $\pi^- K^+$ system can be very different from those in $\pi^- K^-$ one. The systems $\pi^+ K^+$ and $\pi^+ K^-$, which are not discussed here, are analogous to, respectively, $\pi^- K^-$ and $\pi^- K^+$. At first we treat the pions and kaons as a mixture of the ideal gases of π and K . Since the pions and kaons are of a given charge (plus or minus) $g_\pi = g_K = 1$. The masses are taken, respectively, 140 and 494 MeV. Φ as a function of temperature has been computed numerically from the formulas (1,2,3) combined with (8). The results, which are obviously the same for the $\pi^- K^+$ and $\pi^- K^-$ systems, are shown with the dashed lines in Figs. 1-4. The calculations have been performed for several values of the chemical potentials of pions and kaons. In the case of pions, $\mu_\pi = 0$ when the system is in the chemical equilibrium. (We obviously neglect here a tiny effect of the electric charge conservation.) The finite value of μ_K appears even in the equilibrium system of zero net strangeness due to the simultaneous baryon and strangeness conservation. For example, the estimated value of μ_K for the strange (not antistrange) mesons is 38 MeV at $T = 160$ MeV for the equilibrium hadronic system produced in heavy-ion collisions at CERN SPS [6].

In Figs. 1 and 2 we observe a dramatic increase of Φ with the temperature. Φ also grows with μ_K (at $\mu_\pi = 0$) while the dependence on μ_π changes with the temperature. Below $T \cong 120$ MeV Φ is a decreasing function of μ_π but above this temperature Φ grows with μ_π (μ_K is fixed and equals zero). Such a behaviour can be easily understood. Φ can be approximated as

$$\Phi \cong \frac{\sqrt{\langle N_K^2 \rangle - \langle N_K \rangle^2} - \sqrt{\langle N_K \rangle}}{\sqrt{\langle N_\pi \rangle}} , \tag{10}$$

for $\langle N_\pi \rangle \gg \langle N_K \rangle$ which holds for sufficiently low temperatures. (We also assume here that $\langle N_K^2 \rangle - \langle N_K \rangle^2$ is *not* larger than $\langle N_\pi^2 \rangle - \langle N_\pi \rangle^2$.) The growth of μ_K leads to the increase of the numerator of the expression (10) while the growth of μ_π enlarges the denominator. At higher temperatures the numbers of pions and kaons are comparable to each other and the pion dispersion $\langle N_\pi^2 \rangle - \langle N_\pi \rangle^2$ provides a significant contribution to Φ . Then, Φ grows with μ_π .

It is a far going idealization to model a fireball at freeze-out as an ideal gas of pions and kaons. A substantial fraction of the final state particles come from the hadron resonances. We take them into account in the following way. Since the resonances are relatively heavy, their phase-space density is rather low. Consequently, the resonances can be treated as classical particles with the poissonian multiplicity distribution [12]. Then, one easily shows that

$$\langle N_i \rangle = \langle N'_i \rangle + \sum_r b_r \langle N_r \rangle, \quad i = \pi, K \quad (11)$$

$$\langle N_i^2 \rangle - \langle N_i \rangle^2 = \langle N_i'^2 \rangle - \langle N'_i \rangle^2 + \sum_r b_r \langle N_r \rangle,$$

$$\langle N_\pi N_K \rangle - \langle N_\pi \rangle \langle N_K \rangle = \langle N'_\pi N'_K \rangle - \langle N'_\pi \rangle \langle N'_K \rangle + \sum_r b_r^* \langle N_r^* \rangle,$$

where N'_i is the number of ‘direct’ pions or kaons which are described by the formulas (8); the summation runs over the resonances which decay into pions or kaons; N_r is the number of resonances of type r and b_r is the branching ratio of the resonance decay into the pion or kaon channel. The resonances are assumed to decay into no more than one pion or one kaon of interest. We denote with $*$ the resonances, such as K^* , which decay into the pion-kaon pair under study.

In the actual calculations we have taken into account the lightest resonances: $\rho(770)$, $\omega(782)$ and $K^*(892)$. Now, an important difference between the correlations in the $\pi^- K^-$ and $\pi^- K^+$ system appears. The decays of \bar{K}^{*0} into $K^+ \pi^-$ produce the correlation in the $\pi^- K^+$ system. Analogous correlation in the $\pi^- K^-$ system is absent. Φ as a function of temperature has again been computed numerically from the formulas (1,2,3) combined with eqs. (8) which are now supplemented with eqs. (11). The results are shown with the solid lines in Figs. 1 and 2 for the $\pi^- K^-$ system and in Fig. 3 and 4 for the $\pi^- K^+$ one. The calculations have been again performed for several values of the chemical potentials of pions and kaons. The chemical potential of ρ and ω has been taken to be equal to μ_π while that of K^* equals μ_K . One sees that in the case of $\pi^- K^-$ correlations the presence of resonances does not change the results qualitatively although the value of Φ is significantly reduced. The case of $\pi^- K^+$ is changed dramatically due to the resonances. The role of the term corresponding to $\langle N_\pi N_K \rangle - \langle N_\pi \rangle \langle N_K \rangle$ appears to be so important that Φ becomes negative for sufficiently large temperatures. It is somewhat surprising that Φ from Fig. 4 changes its sign at the temperature of about $T = 110$ MeV which is approximately *independent* of μ_K . Such a behaviour can be understood as follows. Since $\langle N_\pi \rangle > \langle N_K \rangle$ in the domain of the parameter values of interest, we expand the expressions (3) and (2) in powers of $\langle N_K \rangle / \langle N_\pi \rangle$. One observes that the first power terms of (3) and (2) cancel out each other. Therefore, $\Phi = 0$ when the second power terms of (3) and (2) are equal to each other. Taking into account that the kaons are approximately classical and consequently Φ depends on μ_K roughly as $e^{\beta \mu_K}$, one indeed finds that the position of $\Phi = 0$ is approximately independent of μ_K .

At the end we take an effort to estimate Φ from the existing experimental data [25] which appear to be rather scarce. Specifically, we consider the system of K_s^0 and negative hadrons produced in pp interactions at the energy 205 GeV, which is close to the currently available energies of heavy-ion collisions at CERN SPS. This case is expected to be similar to the $\pi^- K^+$ system discussed above. One finds in [25] that:

$$\langle N_- \rangle = 2.84, \quad \langle N_-^2 \rangle - \langle N_- \rangle^2 = 3.63$$

$$\langle N_K \rangle = 0.18, \quad \langle N_- N_K \rangle - \langle N_- \rangle \langle N_K \rangle = 0.078.$$

Unfortunately, there are no data on $\langle N_K^2 \rangle - \langle N_K \rangle^2$ which gives a dominant contribution to Φ . If the multiplicity distribution of kaons is poissonian i.e. $\langle N_K^2 \rangle - \langle N_K \rangle^2 = \langle N_K \rangle$, we get $\Phi = -0.004$. The dispersion of the negative hadron multiplicity distribution is known to follow the so-called Wróblewski formula [26]. Applying the formula to the kaons we have $\langle N_K^2 \rangle - \langle N_K \rangle^2 = (0.58 \langle N_K \rangle + 0.29)^2 = 0.16$. In this case the kaon multiplicity appears to be even narrower than the poissonian one and $\Phi = -0.02$. The estimate of $\Phi = 0.007$ given in [22] exceeds the two our numbers because the kaon multiplicity distribution, which is used in [22], is (after averaging over the negative hadron multiplicity) broader than the poissonian one. We conclude that the existing data give a rather poor information on Φ in pp collisions.

We summarize our study as follows. The Φ -measure seems to be a useful tool to study the chemical fluctuations in heavy-ion collisions. If the particles of different species are produced independently from each other and the multiplicity distributions are poissonian, Φ is exactly zero. When the particles are produced in such a way that there are no chemical fluctuations (particle ratios are fixed), Φ is negative but larger than $-1/2$. If the nucleus-nucleus collision is a simple superposition of N-N interactions the value Φ is strictly independent of the collision centrality. The same happens when the hadronic system produced in the nucleus-nucleus collisions achieves the equilibrium with the temperature and chemical potentials being independent of the impact parameter. The thermal model, which seems to be successful in describing the average multiplicities of different particle species, gives a definite prediction of Φ , which is positive for bosons and negative for fermions. The correlations in the system of pions and kaons have been considered in detail. The estimate of Φ for the π^-K^- system is rather reliable while the prediction concerning the π^-K^+ correlations is sensitive to the details of the model. Since the experimental value of Φ in pp interactions can be hardly extracted from the existing data, the fluctuation measurements of nuclear collisions should start with the nucleon-nucleon case.

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Figure Captions

Fig. 1. Φ -measure of the $\pi^- K^-$ correlations as a function of temperature for three values of the pion chemical potential. The kaon chemical potential vanishes. The resonances are either neglected (dashed lines) or taken into account (solid lines). The most upper dashed line on the right hand side of the figure corresponds to $\mu_\pi = 100$ MeV, the central one to $\mu_\pi = 0$, and the lowest line to $\mu_\pi = -100$ MeV. At sufficiently small temperatures the respective dashed and solid lines coincide.

Fig. 2. Φ -measure of the $\pi^- K^-$ correlations as a function of temperature for three values of the kaon chemical potential. The pion chemical potential vanishes. The resonances are either neglected (dashed lines) or taken into account (solid lines). The most upper dashed line corresponds to $\mu_K = 100$ MeV, the central one to $\mu_K = 0$, and the lowest line to $\mu_K = -100$ MeV. At sufficiently small temperatures the respective dashed and solid lines coincide.

Fig. 3. The absolute value of Φ -measure of the $\pi^- K^+$ correlations as a function of temperature for three values of the pion chemical potential. The kaon chemical potential vanishes. The resonances are either neglected (dashed lines) or taken into account (solid lines). The most upper dashed line on the right hand side of the figure corresponds to $\mu_\pi = 100$ MeV, the central one to $\mu_\pi = 0$, and the lowest line to $\mu_\pi = -100$ MeV. At sufficiently small temperatures the respective dashed and solid lines coincide.

Fig. 4. The absolute value of Φ -measure of the $\pi^- K^+$ correlations as a function of temperature for three values of the kaon chemical potential. The pion chemical potential vanishes. The resonances are either neglected (dashed lines) or taken into account (solid lines). The most upper dashed and solid lines correspond to $\mu_K = 100$ MeV, the central ones to $\mu_K = 0$, and the lowest lines to $\mu_K = -100$ MeV. At sufficiently small temperatures the respective dashed and solid lines coincide.







